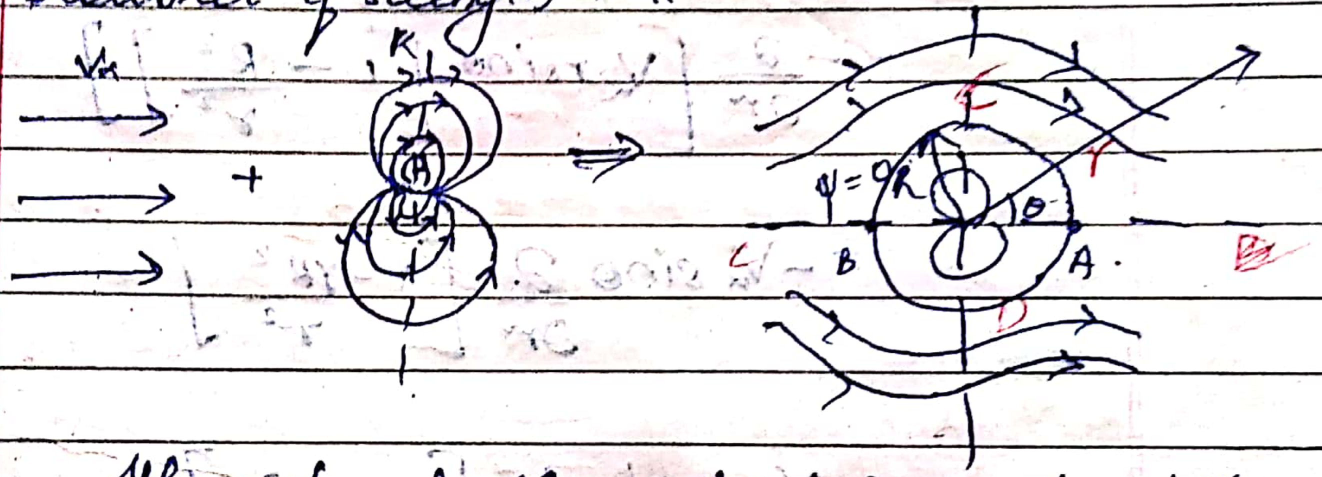
MODULE - 3.

NON LIFTING FLOW OVER CIRCULAR CYLINDER

Consider a uniform flow with velocity V_∞ & doublet of strength K



The streamfunction for the combined flow is

$$\psi = V_\infty r \sin\theta - \frac{K}{2\pi}\frac{\sin\theta}{r}$$

$$\psi = V_\infty \sin\theta \, r\left[1 - \frac{K}{2\pi V_\infty}\frac{1}{r^2}\right]$$

$$\psi = V_\infty r \sin\theta \left[1 - \frac{R^2}{r^2}\right]$$

$$\therefore \frac{K}{2\pi V_\infty} = R^2$$

velocity field is $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$.

$$V_r = V_\infty \cos\theta\left[1 - \frac{R^2}{r^2}\right]$$

$$V_\theta = -\frac{\partial \psi}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left[V_\infty r\sin\theta\left[1 - \frac{R^2}{r^2}\right]\right]$$

$$= -V_\infty \sin\theta \frac{\partial}{\partial r}\left[r - \frac{rR^2}{r^2}\right]$$

$$\therefore V_\theta = -V_\infty \sin\theta\left[1 + \frac{R^2}{r^2}\right]$$

@ stagnation point V_r & $V_\theta = 0$.

$\theta = 0$ & π

$r = R$

$$V_r = V_\infty \cos\theta\left[1 - \frac{R^2}{r^2}\right] = 0$$

$R = r$

$$V_\theta = -V_\infty \sin\theta\left[1 + \frac{R^2}{r^2}\right] = 0.$$

$\theta = 0$ & π.

2 stagnation point located at $(r,\theta) = (R,0)$ & (R,π)

we know that the equation of the streamline passes through both stagnation points is zero. that is $\psi = 0$.

$$\psi = V_\infty r\sin\theta\left[1 - \frac{R^2}{r^2}\right] = 0$$

when $R = r$

$\theta = 0$ & π.

This equation satisfied by $\theta = \pi$ & $\theta = 0$ & $R = r$

The velocity distribution at the surface of the cylinder when $r = R$, $\therefore V_r = 0$ & $V_\theta = -2V_\infty \sin\theta$ $(V_\theta = V)$

$$\because V_\theta = -V_\infty \sin\theta\left[1 + \frac{R^2}{R^2}\right] = -2V_\infty \sin\theta.$$

pressure coefficient $C_p = 1 - \left(\frac{V}{V_\infty}\right)^2$

$$C_p = 1 - \left[-\frac{2V_\infty \sin\theta}{V_\infty}\right]^2$$

$V = V_\theta$

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$C_p = 1 - 4\sin^2\theta$

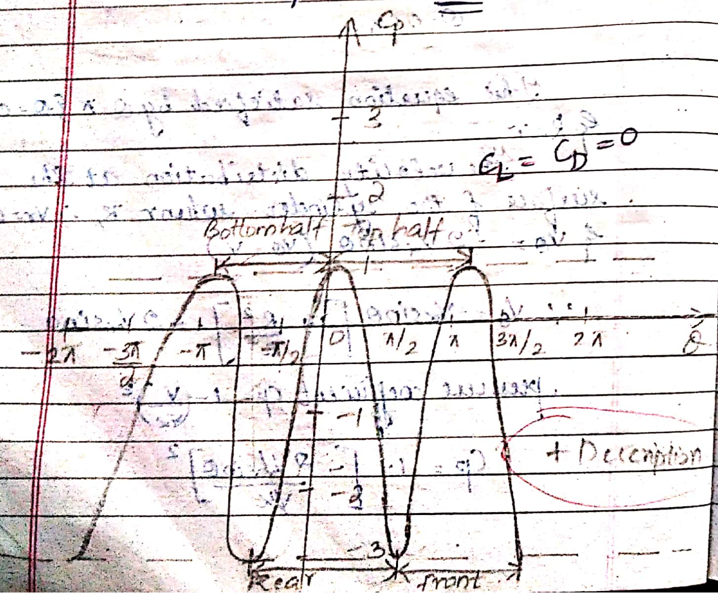
@ stagnation point $\theta = 0, \pi$

so $C_p = 1$.

@ the point of maximum velocity
$\theta = \pi/2$

$C_p = 1 - 4 = -3$

$C_L = C_D = 0$



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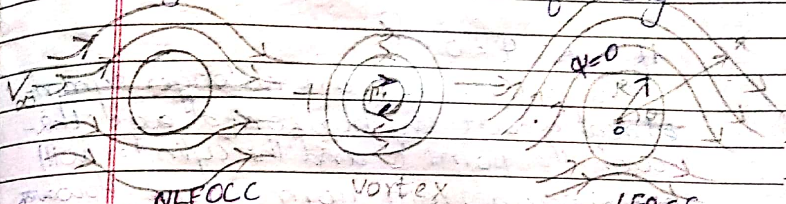
* Lifting flow over a circular cylinder:
Consider the flow synthesized by the addition of the non-lifting flow over a cylinder and a vortex of strength 'Γ'.



The stream function for non-lifting flow over a circular cylinder of Radius 'R' is:

$$\psi_1 = V_\infty r \sin\theta \left[1 - \frac{R^2}{r^2}\right]$$

The stream function for a vortex flow with strength Γ is:

$$\psi_{vor} = \frac{\Gamma}{2\pi} \ln r + \text{constant}$$

$$\psi_{vor} = \frac{\Gamma}{2\pi} \ln r - \frac{\Gamma}{2\pi} \ln R$$

$$\psi_{vor} = \frac{\Gamma}{2\pi} \ln\left(\frac{r}{R}\right)$$

Therefore, total stream function:

$$\psi = V_\infty r \sin\theta \left[1 - \frac{R^2}{r^2}\right] + \frac{\Gamma}{2\pi} \ln\left(\frac{r}{R}\right)$$

If $r = R$, $\psi = 0$.

The streamlines are no longer symmetrical about the horizontal axis through the point 'O' and the cylinder will experience a finite normal force. However the streamlines are symmetrical about the vertical axis through 'O', as a result the drag will be zero.

The velocity field, $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$.

$$V_r = \frac{1}{r} V_\infty \cos\theta \cdot r \left[1 - \frac{R^2}{r^2}\right]$$

$$V_r = V_\infty \cos\theta \left[1 - \frac{R^2}{r^2}\right]$$

$$V_\theta = \frac{-\partial \psi}{\partial r}$$

$$V_\theta = -V_\infty \sin\theta \left[1 + \frac{R^2}{r^2}\right] - \frac{\Gamma}{2\pi r}$$

At stagnation point, $V_r = V_\theta = 0$.

$r = R$

$$V_r = 0 \implies V_\infty \cos\theta \left[1 - \frac{R^2}{r^2}\right] = 0$$

$\therefore R = r$

$$V_\theta = 0 \implies -V_\infty \sin\theta \left[1 + \frac{R^2}{r^2}\right] - \frac{\Gamma}{2\pi r} = 0$$

$$\boxed{r = R} \quad = -V_\infty \sin\theta \left[1 + \frac{R^2}{R^2}\right] - \frac{\Gamma}{2\pi R}$$

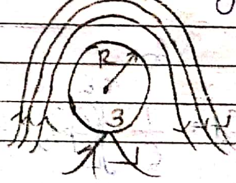
$$= -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R} = 0.$$

$$\sin\theta = \frac{-\Gamma}{2\pi R 2 V_\infty}$$

$$\theta = \sin^{-1}\left[\frac{-\Gamma}{4\pi R V_\infty}\right]$$

If ~~circulation~~ $\frac{\Gamma}{4\pi R V_\infty} < 1$, ~~the result is~~ valid.

2 stagnation points on the bottom half of the circular cylinder.



If $\frac{\Gamma}{4\pi R V_\infty} = 1$.

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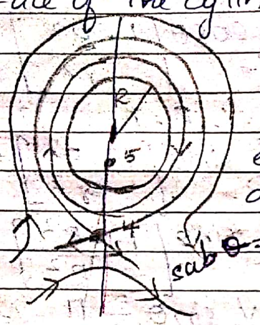
There is only 1 stagnation point on the bottom surface of the cylinder.



If $\frac{\Gamma}{4\pi R V_\infty} > 1$, then the equation has no meaning. in V_θ

sub $\theta = \frac{\pi}{2}$ & $\frac{3\pi}{2}$

Then the velocity on the surface of the cylinder:

$(r = R)$ $V_r = 0$

$$V_\theta = -2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}$$

The pressure coefficient:

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left[\frac{-2V_\infty \sin\theta - \frac{\Gamma}{2\pi R}}{V_\infty}\right]^2$$

$$= 1 - \left[\frac{\frac{\Gamma^2}{4\pi^2 R^2} + 4V_\infty^2 \sin^2\theta + \frac{4V_\infty \sin\theta\,\Gamma}{2\pi R}}{V_\infty^2}\right]$$

Drag coefficient, $C_d = \frac{1}{C}\int_{L.E}^{T.E} \left[(C_p)_u - (C_p)_l\right] dy$

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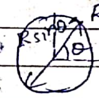
$$= \frac{1}{2R}\int_{\pi}^{0} (C_p)_u R\cos\theta\, d\theta - \frac{1}{2R}\int_{\pi}^{2\pi} (C_p)_l R\cos\theta\, d\theta$$

[where $C = 2R$ and $y = R\sin\theta$, $\therefore dy = R\cos\theta\, d\theta$ then substituting above into first equation]



$$= \frac{1}{2R}\int_{\pi}^{0} C_p R\cos\theta\, d\theta - \frac{1}{2R}\int_{\pi}^{2\pi} C_p R\cos\theta\, d\theta$$

$$= -\frac{1}{2}\int_{0}^{\pi} C_p \cos\theta\, d\theta - \frac{1}{2}\int_{\pi}^{2\pi} C_p \cos\theta\, d\theta$$

[$\because$ limit is given, no need of l and u]

$$C_d = -\frac{1}{2}\int_{0}^{2\pi} C_p \cos\theta\, d\theta$$

Sub the value of C_p in the above equation:

$$C_d = -\frac{1}{2}\int_{0}^{2\pi}\left[1 - \left[\frac{\frac{\Gamma^2}{4\pi^2 R^2} + 4V_\infty^2 \sin^2\theta + \frac{4V_\infty \sin\theta\,\Gamma}{2\pi R}}{V_\infty^2}\right]\right]\cos\theta\, d\theta$$

$\therefore C_d = 0$.

$\therefore$ The drag on a cylinder in an inviscid incompressible flow is zero.

The lift on the cylinder:

Lift coefficient, $C_L = \frac{1}{C}\int_{LE}^{TE} (C_p)_l \, dx$ +

$\frac{1}{C}\int_{LE}^{TE} (C_p)_u \, dx$.

$C = 2R$

$x = R\cos\theta$

$dx = -R\sin\theta \, d\theta$

sub in above eqn:

$$C_L = \frac{1}{2R}\int_{\pi}^{2\pi} -C_p R\sin\theta \, d\theta - \frac{1}{2R}\int_{\pi}^{0} -C_p R\sin\theta \, d\theta$$

$$= -\frac{1}{2R}\int_{\pi}^{2\pi} C_p R\sin\theta \, d\theta + \frac{1}{2R}\int_{\pi}^{0} C_p R\sin\theta \, d\theta$$

$$= -\frac{1}{2R}\int_{\pi}^{2\pi} C_p R\sin\theta \, d\theta - \frac{1}{2R}\int_{0}^{\pi} C_p R\sin\theta \, d\theta$$

$$= -\frac{1}{2}\int_{\pi}^{2\pi} C_p \sin\theta \, d\theta - \frac{1}{2}\int_{0}^{\pi} C_p \sin\theta \, d\theta$$

$$C_L = -\frac{1}{2}\int_{0}^{2\pi} C_p \sin\theta \, d\theta$$

sub the value of C_p in C_L:

$$C_L = -\frac{1}{2}\int_0^{2\pi}\left[1 - \frac{\left[\frac{\Gamma^2}{4\pi^2R^2} + 4V_\infty^2\sin^2\theta + 4V_\infty\sin\theta\frac{\Gamma}{2\pi R}\right]}{V_\infty^2}\right]\sin\theta \, d\theta$$

$$\int_0^{2\pi}\sin^2\theta \, d\theta = \int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{1}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{2\pi}$$

$$= \frac{1}{2}\left[2\pi - \frac{\sin 4\pi}{2}\right] = \pi$$

$$\left[\begin{array}{l}\int_0^{2\pi}\sin\theta \, d\theta = 0 \\ \int_0^{2\pi} 4\sin^3\theta \, d\theta = 0\end{array}\right]$$

$$C_L = -\frac{1}{2}\times\int_0^{2\pi} -\frac{2\Gamma\sin^2\theta \, d\theta}{\pi R V_\infty}$$

$$C_L = -\frac{1}{2}\times -\frac{2\Gamma}{\pi R V_\infty}\times\pi = \frac{\Gamma}{RV_\infty}$$

Lift per unit span, L' is obtained by:

$$L' = \frac{1}{2}\rho_\infty V_\infty^2 S \, C_l \qquad S = 2R\times 1.$$

$$\therefore L' = \frac{1}{2}\rho_\infty V_\infty^2 \, 2R\frac{\Gamma}{RV_\infty} = \rho_\infty V_\infty \Gamma$$

$$\therefore L' = \rho_\infty V_\infty \Gamma \;//$$

Module - III

Magnus force

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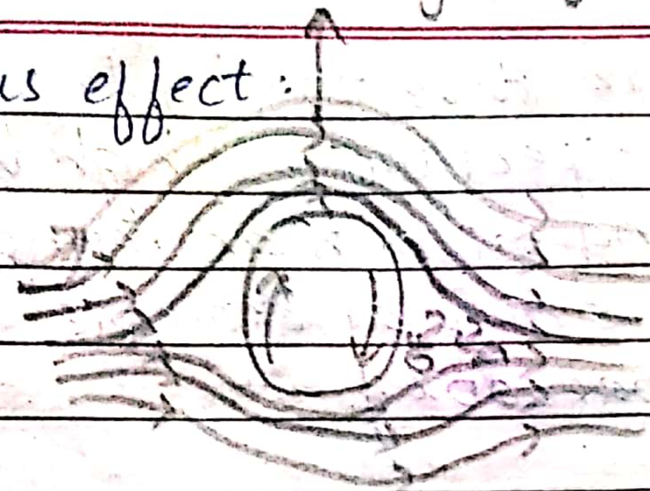
* Magnus effect:



Magnus effect is a phenomenon that is commonly associated with a spining object moving through the air or another fluid. The path of the spining object is deflected in a manner that is not present when the object is not spining. The deflection can be explained by the difference in pressure of the fluid on opposite side of the spining object. The magnus effect depends on the speed of rotation.

* D-Alembert's paradox:

(D Alembert proved that for incompressible & invicid potential flow the drag force is zero on a body moving with constant velocity relative to the fluid) zero drag is in direct contradiction to the observation of substantial drag on bodies moving relative to fluids such as air & water, especially at high velocities corresponding with high reynolds number. It is a particular example of the reversibility paradox.)

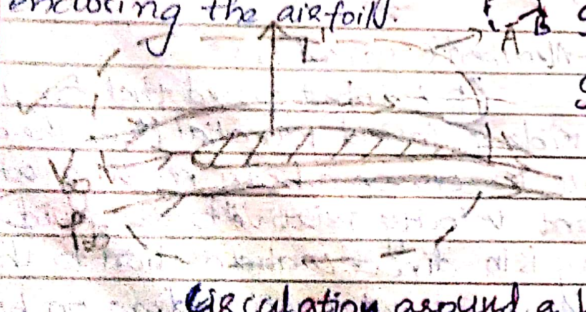
Module-III

Kutta Joukowski Theorem

The Kutta-Joukowski theorem states that lift per unit span on a 2 dimensional body is directly proportional to the circulation around the body.

Consider the incompressible flow over an airfoil section.

Let curve 'A' be any curve in the flow enclosing the airfoil.



$\oint_A V \cdot ds = \Gamma$

$\oint_B V \cdot ds = 0$

Circulation around a lifting airfoil

If the airfoil is producing lift, the velocity field around the airfoil will be such that the line integral of velocity around 'A' will be finite that is, the circulation:

$$\Gamma \equiv \oint V \cdot ds$$

is finite. In turn the lift per unit span L' on the airfoil will be given by Kutta-Joukowski theorem.

$$\boxed{L' = \rho_\infty V_\infty \Gamma}$$

The lifting flow over a circular cylinder was synthesized by superimposing a uniform flow, a doublet and a vortex. All three elementary flows are irrotational at all points except for the vortex, which has infinite vorticity at the origin.

→ Therefore the lifting flow over a cylinder is irrotational at every point except at the origin.

→ If we take the circulation around any curve not enclosing the origin, then $\Gamma = 0$.

→ It is only when we choose a curve that encloses the origin, where $\nabla \times V$ is infinite and yields a finite Γ, equal to the strength of vortex.

→ The same thing will happen about the flow over the airfoil, if the flow outside the airfoil is irrotational and the circulation around any closed curve not enclosing the airfoil is zero.

→ On the other hand, if the flow over an airfoil is sythesized by distributing vortices either on the surface or inside the airfoil, then these vortices have the usual singularities in $\nabla \times V$ and yields a finite value of Γ, equal to the sum of the vortex strengths distributed on or inside the airfoil.

→ In Kutta-Joukowski theorem, the value of Γ must be evaluated around a closed curve that encloses the body; the curve can be otherwise arbitrary, but it must have the body inside it.

→ The Kutta-Joukowski theorem is simply an alternative way of expressing the consequences of the surface pressure distribution of invicid and incompressible flow.

→ Lift is caused by the net imbalance of the surface pressure distribution and the circulation is simply a defined quantity determined from the same pressures. Therefore it is not quite proper to state that circulation causes lift.

→ The relation between the surface pressure distribution (which produces lift L') and circulation is given by eqn: $L' = \rho_\infty V_\infty \Gamma$

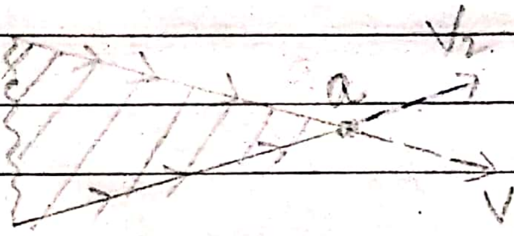
→ Determination of circulation is much easier than calculating the detailed surface pressure distribution.

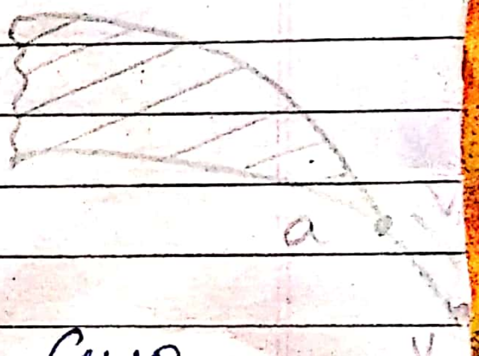
→ Once 'Γ' is obtained, then the lift per unit span follows directly from the Kutta-Joukowski theorem.

Kutta Condition

The steady flow over a given airfoil at a given angle of attack has particular value of circulation which results in the flow leaving smoothly at the trailing edge. This is known as kutta condition.

The trailing edge can have a finite angle or it can be cusped.





finite angle

At point a: $V_1 = V_2 = 0$

Cusp.

At point a: $V_1 = V_2 \neq 0$

Consider the trailing edge with a finite angle. Let the velocities along the top surface and the bottom surfaces as V_1 and V_2 respectively.

V_1 is parallel to the top surface and V_2 is parallel to the bottom surface @ point 'a'.

For the finite trailing edge, if these velocities were finite at point a,

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in a different directions. Point 'a' is the stagnation point where $V_1 = V_2 = 0$.

For the cusped trailing edge, V_1 and V_2 are in same direction at point 'a' and hence both V_1 and V_2 can be finite.

The pressure at point 'a', 'P_2' is a single unique value.

By Bernoulli's Eqn :

$$P_a + \frac{1}{2} \rho V_1^2 = P_a + \frac{1}{2} \rho V_2^2$$

i.e., $V_1 = V_2$

For the cusped trailing edge, the velocities leaving the top and bottom surfaces of the airfoil at the trailing edge are finite and equal in magnitude and direction.

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Summarize the statement of Kutta condition as follows:

(i) for a given airfoil at a given angle of attack the value of Γ around the airfoil is such that the flow leaves the trailing edge smoothly.

(ii) If the trailing edge angle is finite then the trailing edge is a stagnation point.

(iii) If the trailing edge is cusped, then the velocities leaving the top and bottom surfaces at the trailing edge are finite and equal in magnitude and direction.

The statement of Kutta condition in case of vortex sheet is:

$$\gamma(s) = \gamma(T.E) = \gamma(a) = V_1 - V_2$$

Where, $\gamma(s)$ is the strength of vortex sheet.

For finite - angle trailing edge,

$$V_1 = V_2 = 0.$$

i.e., $\gamma(T.E) = 0$

For the cusped trailing edge;

$$V_1 = V_2 \neq 0$$

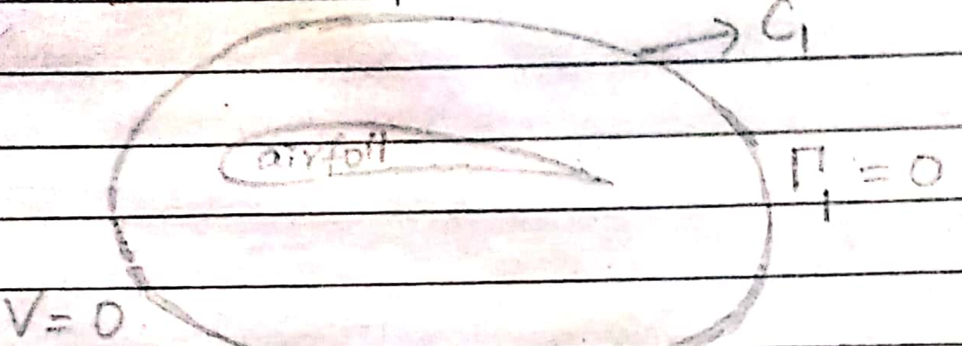
i.e., $\gamma(T.E) = 0.$

Therefore, the Kutta condition expressed in terms of strength of the vortex sheet is:

$$\boxed{\gamma(T.E) = 0.}$$

* Starting vortex:

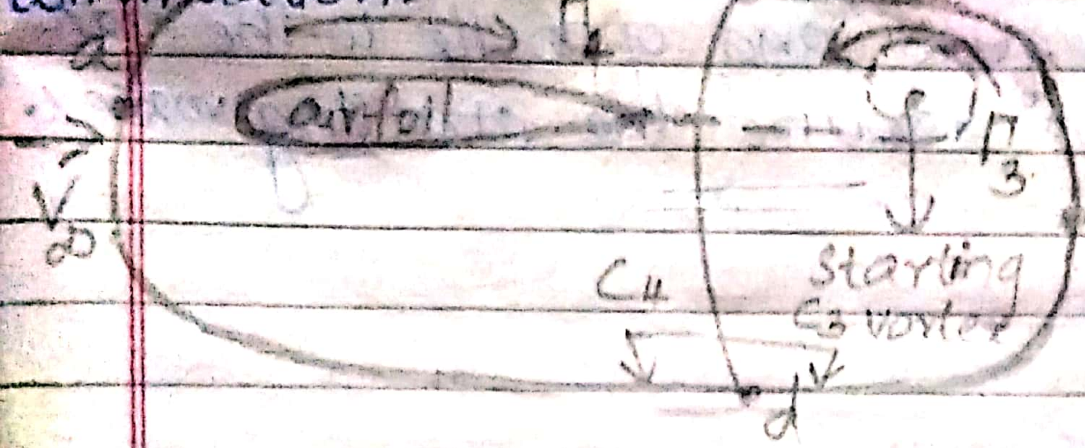
Consider an airfoil in a fluid at rest $V=0$ everywhere and the circulation around curve C_1 is zero.



Fluid at rest relative to airfoil. When the flow in motion over the airfoil, the flow will tend to curl around the trailing edge. The velocity tends towards a very large finite number. During the very first moments after the flow is started, a thin region of very large velocity gradients and therefore high vorticity is formed at the trailing edge.

This high vorticity region is fixed ~~focuse~~ to the same fluid elements and it is flushed downstream as the fluid elements begins to move downstream from the trailing edge.

continuation:



$\Gamma_2 = \Gamma_1 = 0$

$\Gamma_3 = -\Gamma_4$

Some moments after the start of the flow.

The flow field sometime after steady flow has been achieved over the air-foil, with the starting vortex somewhere down stream.

The fluid elements that initially made up curve 'c' and moved downstream and make up curve 'C_2'.

From Kelvin's theorem:

$$\Gamma_1 = \Gamma_2 = 0.$$

Let us subdivide C_2 into 2 loops by making the cut 'bd'; thus forming curves

C_3 and C_4.

$\therefore \Gamma_2 = \Gamma_3 + \Gamma_4$

where $\Gamma_2 = 0$.

$\therefore \Gamma_3 = -\Gamma_4$

The circulation around the airfoil is equal and opposite to the circulation around the starting vortex.